MODEL ANSWERS

Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number

Candidate Number

Further Mathematics

Advanced Subsidiary
Further Mathematics options
23: Further Statistics 1
(Part of options B, E, F and G)

Thursday 17 May 2018 – Afternoon

Paper Reference
8FM0-23

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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P 6 0 1 5 4 A 0 1 1 6

Pearson

Answer ALL questions. Write your answers in the spaces provided.

1. A researcher is investigating the distribution of orchids in a field. He believes that the Poisson distribution with a mean of 1.75 may be a good model for the number of orchids in each square metre. He randomly selects 150 non-overlapping areas, each of one square metre, and counts the number of orchids present in each square.

The results are recorded in the table below.

Number of orchids in each square metre	0	1	2	3	4	5	6
Number of squares	30	42	35	26	11	6	0

He calculates the **expected** frequencies as follows

Number of orchids in each square metre	0	1	2	3	4	5	More than 5
Number of squares	26.07	45.62	39.91	23.28	10.19	3.57	r

(a) Find the value of r giving your answer to 2 decimal places.
(1)

The researcher will test, at the 5% level of significance, whether or not the data can be modelled by a Poisson distribution with mean 1.75

(b) State clearly the hypotheses required to test whether or not this Poisson distribution is a suitable model for these data.
(1)

The test statistic for this test is 2.0 and the number of degrees of freedom to be used is 4

(c) Explain fully why there are 4 degrees of freedom.
(2)

(d) Stating your critical value clearly, determine whether or not these data support the researcher's belief.
(2)

The researcher works in another field where the number of orchids in each square metre is known to have a Poisson distribution with mean 1.5

He randomly selects 200 non-overlapping areas, each of one square metre, in this second field, and counts the number of orchids present in each square.

(e) Using a Poisson approximation, show that the probability that he finds at least one square with exactly 6 orchids in it is 0.506 to 3 decimal places.
(4)

a) $r = 150 - (26.07 + 45.62 + 39.91 + 23.28 + 10.19 + 3.57)$

$= \boxed{1.36}$

Question 1 continued

b) H_0: Po(1.75) is a suitable model for these data.

H_1: Po(1.75) is not a suitable model for these data.

c) last 3 cells need to be added together to ensure expected frequencies are all greater than 5.

There are 5 total cells after pooling so $\nu = 5-1 = \boxed{4}$

(we always subtract 1 from # of cells).

d) $\chi^2_4 = 9.488$

$2.0 < 9.488$ ∴ Result is insignificant. Accept H_0.

Po(1.75) is a suitable model for the data

So the researcher's belief is supported.

e) $P(X=6) = \dfrac{e^{-1.5}(1.5^6)}{6!} = 0.00353$

\# of areas with 6 orchids

∴ $Y \sim B[200, 0.00353] \simeq Po(0.706)$

↑ 200×0.00353

$P(\text{required}) = P(Y \geq 1) = 1 - P(Y=0)$

$= 1 - e^{-0.706} = \boxed{0.506}$

P 6 0 1 5 4 A 0 3 1 6

2. The number of heaters, H, bought during one day from *Warmup* supermarket can be modelled by a Poisson distribution with mean 0.7

(a) Calculate $P(H \geqslant 2)$ $H \sim Po(0.7)$

(1)

The number of heaters, G, bought during one day from *Pumraw* supermarket can be modelled by a Poisson distribution with mean 3, where G and H are independent.

(b) Show that the probability that a total of fewer than 4 heaters are bought from these two supermarkets in a day is 0.494 to 3 decimal places.

(2)

(c) Calculate the probability that a total of fewer than 4 heaters are bought from these two supermarkets on at least 5 out of 6 randomly chosen days.

(3)

December was particularly cold. Two days in December were selected at random and the total number of heaters bought from these two supermarkets was found to be 14

(d) Test whether or not the mean of the total number of heaters bought from these two supermarkets had increased. Use a 5% level of significance and state your hypotheses clearly.

(5)

a) $P(H \geqslant 2) = 1 - P(H \leqslant 1) = 1 - 0.844\ldots = 0.156$ (3dp)

b) $G \sim Po(3)$, $H \sim Po(0.7)$

$(G+H) \sim Po(3.7)$

$P(G+H < 4) = P(G+H \leqslant 3)$

$= \boxed{0.494}$ (3dp)

c) let # of days where fewer than 4 heaters are bought $= X$,

then $X \sim B(6, 0.494)$

$P(\text{required}) = P(X \geqslant 5) = 1 - P(X \leqslant 4)$

$= 1 - 0.896\ldots$

$= \boxed{0.104}$ (3dp)

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Question 2 continued

d) $H_0: \lambda = 3.7$

$H_1: \lambda > 3.7$

let total # of heaters bought for 2 days = Y

$Y \sim Po(7.4)$

$P(Y \geq 14) = 1 - P(Y \leq 13)$

$= 1 - 0.9804...$

$= 0.0195$

$0.0195 < 0.05$

∴ Result is significant. Reject H_0. Evidence suggests that the total # of heaters bought from both supermarkets has increased.

P 6 0 1 5 4 A 0 7 1 6

3. A fair six-sided black die has faces numbered 1, 2, 2, 3, 3 and 4

The random variable B represents the score when the black die is rolled.

(a) Write down the value of E(B)

(1)

A white die has 6 faces numbered 1, 1, 2, 4, 5 and c where $c > 5$
The discrete random variable W represents the score when the white die is rolled and has probability distribution given by

w	1	2	4	5	c
$P(W = w)$	$a + b$	a	0.3	a	b

Greg and Nilaya play a game with these dice.

Greg throws the black die and Nilaya throws the white die. Greg wins the game if he scores at least two more than Nilaya, otherwise Greg loses.

The probability of Greg winning the game is $\frac{1}{6}$

(b) Find the value of a and the value of b
Show your working clearly.

(5)

The random variable $X = 2W - 5$

Given that $E(X) = 2.6$

(c) find the exact value of Var(X)

(6)

a) $E(B) = \frac{(1+2+2+3+3+4)}{6} = \boxed{2.5}$

b) Greg wins if: $G = 4$ & $W = 1$ or 2

$G = 3$ & $W = 1$

$P(G=4 \cap W=1) = (\frac{1}{6})(a+b)$

$P(G=4 \cap W=2) = (\frac{1}{6})(a)$

$P(G=3 \cap W=1) = (\frac{2}{6})(a+b)$

$P(\text{Greg wins}) = \frac{1}{6}$

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Question 3 continued

$\therefore \frac{1}{6}(a+b) + \frac{1}{6}a + \frac{2}{6}(a+b) = \frac{1}{6}$

$\Rightarrow a+b+a+2a+2b = 1$

$\Rightarrow 4a+3b = 1$ // — ①

and using $\sum P(W=w) = 1$:

$\therefore a+b+a+0.3+a+b = 1$

$\Rightarrow 3a+2b = 0.7$ // — ②

solving ① & ② simultaneously:

① : $a = \frac{1-3b}{4} \longrightarrow$ ② : $3\left(\frac{1-3b}{4}\right)+2b = 0.7$

$\Rightarrow \frac{3}{4}(1) - \frac{9}{4}(b) + 2b = 0.7$

$\Rightarrow \frac{3}{4} - \frac{b}{4} = 0.7 \quad \therefore \boxed{b = 0.2}$

and $a = \frac{1-3(0.2)}{4} = \boxed{0.1} = a$

CHECKING

$0.1+0.2+0.1+0.3+0.1+0.2 = 1$

$X = 2W - 5$

Question 3 continued

c) $E(X) = E(2W-5) = 2E(W) - 5 = 2.6$

$E(W) = \sum wP(W=w)$

$= 1(a+b) + 2(a) + 4(0.3) + 5a + cb$

$= 2.2 + 0.2c$

$E(X) = 2(2.2 + 0.2c) - 5 = 2.6$

$\Rightarrow \quad 4.4 + 0.4c = 7.6$

$\therefore \quad c = \frac{7.6 - 4.4}{0.4} = 8$ //

$Var(X) = Var(2W-5) = 2^2 Var(W) = 4Var(W)$ //

$E(W) = 2.2 + 0.2c = 2.2 + 0.2(8)$
$= 3.8$ //

$E(W^2) = 1^2(0.3) + 2^2(0.1) + 4^2(0.3) + 5^2(0.1)$
$+ 8^2(0.2) = 20.8$ //

$Var(W) = E(W^2) - [E(W)]^2$

$= 20.8 - (3.8)^2$

$= 6.36$

$\therefore Var(X) = 4(6.36) = \boxed{25.44}$

4. Abram carried out a survey of two treatments for a plant fungus. The contingency table below shows the results of a survey of a random sample of 125 plants with the fungus.

		Treatment		
		No action	Plant sprayed once	Plant sprayed every day
Outcome	Plant died within a month	15	16	25
	Plant survived for 1 – 6 months	8	25	10
	Plant survived beyond 6 months	7	14	5

Abram calculates expected frequencies to carry out a suitable test. Seven of these are given in the partly-completed table below.

		Treatment		
		No action	Plant sprayed once	Plant sprayed every day
Outcome	Plant died within a month			17.92
	Plant survived for 1 – 6 months	10.32	18.92	13.76
	Plant survived beyond 6 months	6.24	11.44	8.32

The value of $\sum \frac{(O-E)^2}{E}$ for the 7 given values is 8.29

Test at the 2.5% level of significance, whether or not there is an association between the treatment of the plants and their survival. State your hypotheses and conclusion clearly.

(7)

H_0: There is no association between the treatment of plants and their survival.

H_1: There is an association between the treatment of plants and their survival.

		Treatment			
		No action	Plant sprayed once	Plant sprayed every day	Total
Outcome	Plant died within a month	15	16	25	56
	Plant survived for 1 – 6 months	8	25	10	43
	Plant survived beyond 6 months	7	14	5	26
	Total	30	55	40	125

$E_i = \frac{\text{row total} \times \text{column total}}{\text{grand total}}$

Question 4 continued

$\therefore E_1 = \frac{56 \times 30}{125} = 13.44$ //

$E_2 = \frac{56 \times 55}{125} = 24.64$ //

		Treatment		
		No action	Plant sprayed once	Plant sprayed every day
Outcome	Plant died within a month	13.44	24.64	17.92
	Plant survived for 1 – 6 months	10.32	18.92	13.76
	Plant survived beyond 6 months	6.24	11.44	8.32

$\therefore \sum \frac{(O-E)^2}{E} = 8.29 + \frac{(15-13.44)^2}{13.44} + \frac{(24.64-16)^2}{24.64}$

$= 11.50$ //

$\nu = (\text{rows}-1)(\text{columns}-1) = (3-1)(3-1) = 4$ //

$\therefore$ critical value $= \chi^2_4 (2.5\%) = 11.143$ //

$11.50 > 11.143$

$\therefore$ Result is significant ; reject H_0.

Evidence suggests there is an association between treatment of plants & their survival.